The Role of Channel Quality in Optimal Allocation of Acquisition and Retention Spending

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Abstract

Optimal promotion budget allocation between acquisition and retention spending is an important topic in the field of customer equity management. We extend this literature in several ways. Most notably, we motivate “channel quality” as a new decision variable to account for the negative relationship between acquisition rate and retention rate. We also address potential non-concavity of the acquisition and retention spending response curves by introducing the ADBUDG function (Little, 1970), which allows for both S-shaped and strictly concave relationships. Further, we relax the assumption in the literature that acquisition rate and retention rate are zero when there is no spending on acquisition and retention, thus accommodating alternative sources of acquisition and retention. Using the decision calculus approach, we apply the model to a prototype real-world problem and provide sensitivity analyses with respect to channel quality, promotion budget, marginal contribution, discount rate, and inaccuracy in the managerial inputs.

Keywords: Customer equity; Promotion budget allocation; Acquisition; Retention; Channel quality; Decision calculus
1. Introduction

There has been a significant shift in the field of marketing from a product- to a customer-oriented focus as marketing managers and researchers recognize that long-term, quality customer-company relationships can be a major source of profitability. Thus, relationship marketing has emerged as an important area of research (e.g., Berger and Nasr, 1998; Rust, Lemon, and Zeithaml, 2004). During this transition, customer equity has become a mainstay concept in the literature, while maximization of customer equity has become a core objective of customer-company relationship management (Berger and Nasr-Bechwati, 2001; Blattberg and Deighton, 1996; Drosch and Carlson, 1996; Venkatesan and Kumar, 2004).

Customer equity has been defined as “…the present value of the expected benefits (e.g., gross margin) less the burdens (e.g., direct costs of servicing and communicating) from customers” (Dwyer, 1997). Thus, effective management of customer equity entails an understanding of the factors, and the interactions among them, which contribute to customer equity (Desai and Mahajan, 1998; Fam and Yang, 2006; Reinartz, Thomas, and Kumar, 2005). One important challenge in this area pertains to the allocation of resources across acquisition and retention activities. To help address this challenge, researchers have developed a number of models. For example, Blattberg and Deighton (1996) examined the issue of optimal expenditures on customer acquisition and customer retention. However, they did not consider acquisition and retention simultaneously. Berger and Nasr-Bechwati (2001) extended Blattberg and Deighton’s (1996) framework and proposed a general approach for examining how a preset (fixed) promotion budget should be allocated between acquisition and retention.
However, a review of the literature suggests several issues remain untreated (Neslin et al., 2006). First, it is often observed that a company which enjoys a higher acquisition rate usually suffers a lower retention rate in the coming purchase cycle. However, extant models have not directly addressed this phenomenon. Second, existing customer equity models assume that both acquisition rate and retention rate are concave with promotion spending. Yet, several studies on advertising and promotion demonstrate that the sales response function can be S-shaped (e.g., Little, 1979; Rao and Miller, 1975). Third, prior models have assumed that acquisition rate and retention rate are zero when no acquisition or retention spending occurs, respectively. However, even in the absence of promotion spending, customers can still obtain information about a product (e.g., via word of mouth) and, for a whole host of reasons, ultimately repurchase it (Blattberg, Getz, and Thomas, 2001; Hogan, Lemon, and Libai, 2004).

We address these gaps in the literature as follows. First, we provide a brief review of a basic model of promotion budget allocation for maximizing customer equity. Consistent with prior research, we use the term “promotion” broadly to include all of the marketing promotion activities a firm initiates for customer acquisition and retention. Next, we extend this basic model by introducing the concept of “channel quality” to address the negative relationship between acquisition rate and retention rate, incorporating the ADBUDG function to accommodate both concave and S-shaped responses (Little, 1970), and adding intercept parameters to allow for non-zero acquisition or retention rate. We then illustrate the use of the new model for a prototype real-world decision problem and provide sensitivity analyses with respect to promotion channel quality, promotion budget, marginal contribution per transaction, discount rate, and inaccuracy in the managerial inputs.
2. Basic customer equity model

Customer equity models can be grouped by the type of buyer-seller relationship. Jackson (1985) proposed two kinds of relationships in industrial buying: always-a-share and lost-for-good. In always-a-share relationships, customers rely on several vendors, giving each a portion of the purchase basket. Thus, customers may frequently switch between vendors. In lost-for-good relationships, customers emphasize long-term commitment when experimenting with the focal vendor due to high switching costs. Thus, when a customer decides to switch, the focal vendor loses the account forever, though the customer may re-appear later as a “new” customer.

Applying Jackson’s (1985) taxonomy, Dwyer (1997) proposed two types of models to capture the lifetime value of customers. Retention models are used to address lost-for-good customers. In these models, the probability that a customer purchases from the focal vendor in the coming purchase cycle is estimated as a function of retention rates, thus emphasizing historical customer relationships. Migration models are used to address always-a-share customers. These models focus on recent customer transitions between vendors rather than historical customer relationships when estimating the probability that a customer purchases from the focal vendor in the coming purchase cycle. Thus, although migration models value customers through various methods, such as cohort separation (e.g., Kumar, Ramani, and Bohling, 2004) and “state” in a Markov chain approach (e.g., Pfeifer and Carraway, 2000; Rust, Lemon, and Zeithmal, 2004), they do not distinguish between acquisition and retention and, thus, provide less ground for discussing promotion budget allocation than retention models.

Blattberg and Deighton (1996) built an influential retention model that examined the optimal spending on acquisition and retention. However, they regarded the two processes as independent and, therefore, did not address how fixed resources should be optimally allocated
between acquisition and retention to maximize customer equity. Building on Blattberg and Deighton’s (1996) work, Berger and Nasr-Bechwati (2001) were the first to propose a quantitative approach for such an allocation problem. They proposed the following model (reaching this form after extensive algebra):

\[
\text{Customer equity} = \max_{A, R} \left( am - A + \frac{a}{1 + d - r} (mr - R) \right),
\]

subject to the following constraints:
1. \( A + aR = B \)
2. \( A \geq 0 \)
3. \( R \geq 0 \)

Here, \( a \) is the acquisition rate, \( A \) is the acquisition spending per prospect, \( m \) is the marginal contribution per transaction, \( R \) is the retention spending per customer, \( r \) is the retention rate, \( d \) is the discount rate appropriate for marketing investments over the purchase cycle, and \( B \) is the preset promotion budget per prospect. Moreover, Berger and Nasr-Bechwati (2001) defined the acquisition rate, \( a \), and the retention rate, \( r \), as follows:

\[
a = C_a (1 - e^{-k_1 A})
\]

\[
r = C_r (1 - e^{-k_2 R})
\]

where \( C_a \) and \( C_r \) are the ceiling rates, and \( k_1 \) and \( k_2 \) are positive constants. This approach has proven effective under a number of different conditions (cf. Berger and Nasr-Bechwati 2001).

In the next section, we extend the basic model. The primary extension is the introduction of channel quality as a decision variable, thereby conceptualizing the negative relationship between acquisition rate and retention rate. Additionally, we address the potential non-concavity of the acquisition and retention spending response curves by introducing the \( ADBUDG \) function (Little, 1970), which allows for both S-shaped and strictly concave relationships, depending on parameter values. Lastly, we relax the assumption in the literature that acquisition rate and
retention rate are zero when there is no spending on acquisition and retention, thus accommodating alternative sources of acquisition and retention.

3. Extended customer equity model

3.1. Channel quality as a decision variable

Wang and Spiegel (1994) argued that acquisition channels can be described in terms of “quality-” versus “quantity-orientation.” When an acquisition channel is relatively strong at acquisition but relatively weak at retention, it is said to be “quantity-oriented,” because acquired customers tend to switch away relatively quickly. When a channel is relatively weak at acquisition but relatively strong at retention, it is said to be “quality-oriented,” because the acquired customers tend to remain with the focal vendor longer.

Consistent with this conceptualization, certain channels seem inherently high quality (e.g., personal selling) while others seem inherently low quality (e.g., price-oriented direct mailings). However, channel characteristics alone do not determine channel quality (Blattberg, Getz, and Thomas, 2001; Bolton, Lemon, and Verhoef, 2004; Gounaris, 2005; Verhoef and Donkers, 2005; Villanueva, Yoo, and Hanssens, 2006). Acquisition channels may also differ in their impact on channel quality if customers who use (or are targeted via) certain channels differ from one another in relevant ways (Bolton, Lemon, and Verhoef 2004; Reinartz, Thomas, and Kumar 2005). For example, since customers generally initiate contact in an Internet channel, they may be easier to retain due to pre-existing awareness and interest than customers who receive an unsolicited direct mailing. Additionally, channel quality can vary due to differences in the products (e.g., low versus high complexity) and messages (e.g., rational versus emotional appeals) that appear in channels (Verhoef and Donkers 2005).
Other empirical evidence is also consistent with Wang and Spiegel’s (1994) assertion that acquisition channels vary meaningfully in terms of quality. Reinartz, Thomas, and Kumar (2005) examined the effects of three company-initiated marketing campaign channels (personal selling, telephone, and email) and one customer-initiated contact channel (Internet) on acquisition and retention for a high tech manufacturer. The results indicated that the frequency of customer contact in each channel was positively related to acquisition and retention. Specifically, the relationship was strongest for personal selling and the Internet, followed by telephone and email, respectively. Further, Jones, Busch, and Dacin (1998) found that customers’ propensity to switch suppliers in a business-to-business service industry was lower when salespersons had more customer-oriented attitudes. Verhoef and Donkers (2005) analyzed the data from a financial-services provider that uses a number of different acquisition channels in various configurations for its services (e.g., auto insurance, housing insurance, health insurance, and loans). They found that the acquisition channels significantly differed in their effect on retention for each type of service, and that these differences depended on the type of service provided.

In sum, there are theoretical and empirical reasons to consider channel quality as a managerial decision variable. We also note that channel quality can be a complex function of characteristics of the channel, customer, product, and message. Thus, our model assumes only that channels differ in quality and that managers can assess the quality of their channels.

3.2. Channel quality and the non-independence of acquisition and retention

Customer acquisition and customer retention are not independent (Thomas, 2001). When firms use a preset promotion budget, spending less on acquisition allows for the possibility of spending more on retention, and vice versa. Since the acquisition rate and retention rate are both
positively associated with promotion spending, it may seem “obvious” that these rates would be negatively related to each other. However, this reasoning ignores the role of channel quality. Specifically, we note that prior research has implied that promotion channel quality contributes to the negative relationship between acquisition rate and retention rate by having a differential impact on the ceiling rates of acquisition and retention (Berger and Nasr-Bechwati, 2001; Wang and Spiegel, 1994). That is, when a higher quality promotion channel is used, the ceiling rate of acquisition, \( C_a \), should be lower, while the ceiling rate of retention, \( C_r \), should be higher. Thus, we associate ceiling rates with promotion channel quality, \( \theta \), in the following way:

\[
C_a = \alpha_0 + \alpha_i \theta^{\alpha_i},
\]

and

\[
C_r = 1 - \beta_j \theta^{\beta_j},
\]

where \( \theta > 0 \), \( \alpha_i > 0 \), and \( \beta_j > 0 \) (\( i = 0, 1, 2; j = 1, 2 \)). We define channel quality such that a channel with a lower \( \theta \) is more quality-oriented. The parameter \( \alpha_0 \) is the ceiling rate of acquisition of a totally quality-oriented promotion channel (i.e., \( \theta = 0 \)), while \( \alpha_i \) and \( \beta_j \) are constants that represent the impact of other factors on acquisition and retention rates, respectively. For different products, the values of \( \alpha_i \) and \( \beta_j \) will differ. Real-world data is used to determine the precise values of the parameters in the functions and, thus, to determine the shapes of the functions (i.e., concave vs. convex).

3.3. Shape of acquisition and retention response functions

Since Blattberg and Deighton (1996), customer equity retention models have assumed that the acquisition rate and the retention rate are (increasing) concave functions of promotion expenditure. However, there has been some debate in the literature about whether the shape of
sales-advertising response is concave or S-shaped (e.g., Johansson, 1979; Little, 1970). The theoretical root of the concavity argument lies in the “law” of diminishing returns to product input in economics (e.g., Stigler, 1961). In the context of advertising, an increasing concave response means that each extra dollar spent on advertisement engenders less and less buying of the target product or service. A number of empirical studies have supported this assertion (e.g., Aaker and Carman, 1982; Ray and Sawyer, 1971; Simon and Arndt, 1980).

However, other empirical studies have suggested that the sales-advertising function can be S-shaped. For instance, Little (1970) proposed that the relationship between brand share and advertising expense is S-shaped. Two widely discussed cases with S-shaped response functions are flight and pulses in an advertising schedule. Little (1979) pointed out that in the two cases, a small advertising rate (i.e., advertising spending per capita) generally does little good, but a medium rate is effective. Empirical evidence supports a general S-shaped relationship between sales and advertising (e.g., Rao and Miller, 1975). Going beyond the context of advertising, an S-shaped curve also appears in the promotion literature. For example, Little’s (1975) BRANDAID model proposed that promotion amplitude and promotion intensity are related in an S-shaped way. With these empirical findings, numerous models have been built to depict the S-shaped relationship, among which one of the most well-known is the ADBUDG function (Little, 1970). Although this function was originally conceived to capture the relationship between brand share and advertising, it can be extended to depict the sales-promotion expense relationship in general.

Thus, we define the acquisition rate and the retention rate as follows:

\[ a = C_a \frac{A^h}{K_1 + A^h} \]  \hspace{1cm} (6)

and

\[ r = C_r \frac{R^h}{K_2 + R^h} \]  \hspace{1cm} (7)
where \( K_1, K_2, b_1, \) and \( b_2 \) are positive constants. These functions have the key property of being concave or S-shaped, depending on the values of \( b_1 \) and \( b_2 \), respectively. Further, as alluded to earlier, these values are determined by real-world data. When \( b_1 > 1 \) and \( b_2 > 1 \), equations (6) and (7) are S-shaped, respectively; otherwise they are concave.

### 3.4. The extended model

As mentioned earlier, existing customer equity maximization models assume acquisition rate and retention rate are zero when no promotion activity occurs. However, when proposing the ADBUDG model, Little (1970) argued that there is a minimum brand share at the point of no advertising. A similar case can be made for acquisition rate and retention rate and, thus, we modify equations (6) and (7):

\[
a = a_0 + (C_a - a_0) \frac{A^h}{K_1 + A^h}
\]

(8)

and

\[
r = r_0 + (C_r - r_0) \frac{R^b_r}{K_2 + R^b_r}
\]

(9)

where \( a_0 \) and \( r_0 \) are the (positive) acquisition rate and retention rate, respectively, at the point of no promotion spending; \( C_a \) and \( C_r \) are the ceiling rates of acquisition and retention defined in equations (4) and (5). Since the ADBUDG function can be used to depict a concave or an S-shaped relationship, the concavity models of Berger and Nasr-Bechwati (2001) and Blattberg and Deighton (1996) can be merged into a model that incorporates the ADBUDG function, channel quality, and non-zero acquisition/retention rates for zero spending. Thus, we define the extended customer equity model as:
Customer equity = \max_{A,R} am - A + \frac{a}{1+d-r} (mr - R), \hspace{1cm} (10) 

subject to the following constraints:
1. \( A + aR \leq B \)
2. \( A \geq 0 \)
3. \( R \geq 0 \)

where:

\[ a = a_0 + (C_a - a_0) \frac{A_h}{K_1 + A_h}, \quad r = r_0 + (C_r - r_0) \frac{R_{h2}}{K_2 + R_{h2}}, \]

\[ C_a = \alpha_0 + \alpha_1 \theta^{\alpha_2}, \quad C_r = 1 - \beta_1 \theta^{\beta_2}. \]

4. Decision calculus

Decision calculus is an approach to model building that incorporates judgments and estimates provided by the decision maker (Little, 1970). It begins with a researcher breaking a complex problem down into smaller parts and devising a set of simple questions pertaining to these parts. A manager then provides answers to the questions and the researcher uses the answers to build a formal model capable of addressing the original problem of interest. One benefit of this method is that it can encourage model usage by managers who hesitate to do so unless they feel a model is an extension of their own experience or analytical repertoire. For example, Little (1970) considered the problem of modeling sales response as a function of advertising expense. In this example, the researcher specified an empirically-based functional form linking sale response and advertising expense. Next, managers were asked simple questions that allowed for estimation of the model parameters (e.g., a manager was asked what their brand share would be at the end of one period if advertising expense was set at a particular level).

In sum, decision calculus allows us to model the complex problem of optimal promotion budget allocation in the context of acquisition and retention response functions while also
engaging the end-user of the resulting model. This is an important feature, as empirical evidence from field studies and laboratory experiments suggests that managers who use support systems based on decision-calculus generally make better decisions than their counterparts not using such models (e.g., van Bruggen, Smidts, and Wierenga, 2001; Lodish, Curtis, Ness, and Simpson, 1988). However, since estimates of the model parameters depend on values estimated by managers, we test the sensitivity of the model to (1) variation in (true) managerial input values and (2) inaccuracy in managerial input values (cf. Chakravarti and Staelin, 1981).

5. A prototype real-world example

5.1. Optimal promotion budget allocation and customer equity

We now present a prototype real-world example to illustrate the model (cf. Berger and Nasr-Bechwati 2001). We introduce the example in stages, first omitting channel quality as a consideration, then building on the results and introducing channel quality.

Suppose an insurance company targets a prospect base of 1,000,000 persons and is now considering allocating its preset promotion budget of $60,000,000 between acquisition and retention for a particular product (note that $B = 60$). Customers buy the insurance product once a year. Assume that the yearly marginal contribution per customer is $m = 400$ and that the appropriate yearly discount rate is $d = 0.20$ (i.e., 20%).

We begin by asking the manager several simple questions about their product promotion in order to derive the parameter values in the extended model and to determine the shapes of acquisition curve and retention curves. For example, to obtain currently used channel quality ($\theta$), we ask the manager to rate the channel quality on an 11-point scale ($0 = \text{totally quality-oriented}$,
10 = totally quantity-oriented). Assume the manager judges the quality of the currently used
channel to be a “6” (i.e., $\theta = 6$). Thereafter, we ask several questions based on the current
promotion channel. First, to get the value of the ceiling rate of acquisition, we ask the manager to
report the acquisition rate that can be achieved in the target market using this channel if there
were no limit on acquisition spending. Assume the answer is 60% (i.e., $C_a = 0.60$). We then ask
the manager to estimate the acquisition rate that can be achieved using this channel if no
acquisition dollars were spent on the target market. Let us assume the answer is 1% (i.e., $a_0 =
0.01$). By asking similar questions about retention, we can obtain the values of the ceiling rate of
retention (assume 65%) and the retention rate with no retention spending (assume 5%). Thus, $C_r
= 0.65$ and $r_0 = 0.05$.

To determine the values of the other parameters ($b_1, b_2, K_1$, and $K_2$), we ask the
managers a few more questions. First, we ask the manager to report the current acquisition
expenditure per prospect and the corresponding acquisition rate. Let us assume the manager
reports $10 and 10%, respectively. Thus, we have:

$$a_{s10} = 0.01 + (0.60 - 0.01)\frac{10^{b_1}}{K_1 + 10^{b_1}} = 0.10$$ (11)

Next, we ask the manager to estimate the acquisition rate if the acquisition spending per prospect
were $10 greater (i.e., $20). Assuming the answer is 25%, we get:

$$a_{s20} = 0.01 + (0.60 - 0.01)\frac{20^{b_1}}{K_1 + 20^{b_1}} = 0.25$$ (12)

Solving equations (11) and (12) simultaneously, we get $K_1 = 472.43$ and $b_1 = 1.93$. Thus, in this
example, the relationship between acquisition rate and acquisition spending happens to be S-
shaped (since $b_1 > 1$; see Figure 1).
Similarly, assume that when asked about the current retention spending and retention rate, the manager reports $20 and 33%, respectively. Thus, we have:

\[ r_{s20} = 0.05 + (0.65 - 0.05) \frac{20^{b_2}}{K_2 + 20^{b_2}} = 0.33 \]  \hspace{1cm} (13)

The manager is also asked to estimate the retention rate if the retention spending per customer were $10 greater (i.e., $30). Let us say the manager reports 50%. We have:

\[ r_{s30} = 0.05 + (0.65 - 0.05) \frac{30^{b_2}}{K_2 + 30^{b_2}} = 0.50 \]  \hspace{1cm} (14)

Solving equations (13) and (14) simultaneously, we get \( K_2 = 10,271.23 \) and \( b_2 = 3.04 \). Hence, the retention rate is also S-shaped with retention spending (see Figure 1; see also Table 1 for a summary of the input values provided by the manager for this example).

Using the parameter values above, along with the managerial inputs, we proceed to find the optimal promotion allocation between acquisition and retention using the model defined in equation (10). Because acquisition and retention spending (i.e., \( A \) and \( R \)) are nonlinearly related to acquisition rate and retention rate (see equations (8) and (9)), the equations generated using an algebraic (e.g., Lagrangian) approach to solve for an optimal solution are intractable. Thus, we use numerical optimization procedures and find that the optimal acquisition spending per prospect \( (A^0) \) is $40.48, optimal retention spending per customer \( (R^0) \) is $44.44, and maximum customer equity \( (CE^0) \) is $275.81. Note that the budget constraint is satisfied since \( A^0 + a^0R^0 = \)
$40.48 + (0.439)(44.44) = 60$, where \( a^0 \) is the acquisition rate at the optimal acquisition spending.

6. Determination of optimal channel quality

Note that the optimal allocation strategy and maximum customer equity values calculated in the previous section are both based on the manager’s currently used promotion channel (i.e., \( \theta = 6 \)). An important implication of our extended model is that the manager, operating under the same budget constraint, can obtain even higher customer equity by optimizing on channel quality. Since channel quality impacts optimal customer equity through its relationships with the ceiling rates of acquisition and retention, we must obtain additional managerial inputs for equations (4) and (5) in order to optimize channel quality.

To get the values of the parameter \( \alpha \) in equation (4), we ask the manager to estimate the ceiling rate of acquisition if the channel were totally quality-oriented. Assume the manager provides an answer of 1% (i.e., \( \alpha = 0.01 \)). Using the previously reported ceiling rates for acquisition (0.60) and retention (0.65), we have:

\[
C_{a|\theta=6} = 0.01 + \alpha_1 \cdot 6^\alpha = 0.60 \tag{15}
\]

\[
C_{r|\theta=6} = 1 - \beta_1 \cdot 6^\beta = 0.65 \tag{16}
\]

To acquire an additional point on each ceiling rate curve, we ask the manager to estimate the ceiling acquisition rate and retention rate if the channel were totally quantity-oriented (i.e., \( \theta = 10 \)). Assume the manager indicates these rates are 80% and 10%, respectively. We have:

\[
C_{a|\theta=10} = 0.01 + \alpha_1 \cdot 10^\alpha = 0.80 \tag{17}
\]

\[
C_{r|\theta=10} = 1 - \beta_1 \cdot 10^\beta = 0.10 \tag{18}
\]
Solving equations (15), (16), (17), and (18) simultaneously, we find $\alpha_1 = 0.2119$, $\alpha_2 = 0.5714$, $\beta_1 = 0.0127$, and $\beta_2 = 1.8489$. Figure 2 illustrates the relationships among $C_a$, $C_r$, and $\theta$.

As mentioned before, $\alpha_1$, $\alpha_2$, $\beta_1$, and $\beta_2$ are assumed to be constant parameters across all promotion channels for this particular product. Using these parameter values, equation (10), and numerical optimization, we find that the optimal channel quality is $\theta = 2.8$. If the manager opts for a promotion channel with this optimal quality, the optimal budget allocation becomes $A = $41.85 per prospect and $R = $62.05 per customer. The new optimal customer equity is $345.00, an increase of $61.19 (or 21.6%) over that obtained with the currently used channel (i.e., $\theta = 6$).

7. Sensitivity analysis

In this section we provide two distinct kinds of sensitivity analysis. The first examines, in a traditional sense, the sensitivity of the optimal values of the decision variables and optimal customer equity to differences in the values of various input parameters. The second, introduced since we are using decision calculus and the manager’s subjective estimates, is to study the sensitivity of the optimal customer equity to the inaccuracy of the manager’s inputs.

7.1. Sensitivity analyses for selected parameter values

We first perform a sensitivity analysis with regard to channel quality ($\theta$). Table 2 shows that when the promotion channel becomes more quantity-oriented (i.e., channel quality increases), the optimal acquisition rate ($a^0$) increases while the optimal retention rate ($r^0$)
decreases. The values of \( C_a \) and \( C_r \) in Table 2 also illustrate the role of channel quality in accounting for this negative relationship. Additionally, Figure 3 depicts the complex relationship between channel quality and both acquisition and retention spending. Finally, consistent with our assertions, Table 2 and Figure 3 indicate that there exists an optimal channel quality in customer equity maximization (as customer equity is \( n \)-shaped with channel quality.

We next conduct a sensitivity analysis with regard to preset promotion budget \((B)\), with the assumption that the manager optimizes the promotion channel (i.e., \( \theta = 2.8 \)). According to Table 3, holding all else constant, an increased preset promotion budget, not surprisingly, results in higher optimal acquisition and retention spending. This leads to increased acquisition rate and retention rate and, therefore, customer equity is increased. For example, when the preset promotion budget increases from $60 to $80 per prospect, optimal acquisition spending increases from $41.04 to $56.27 per prospect and optimal retention spending increases from $57.83 to $63.80 per customer. The optimal customer equity increases from $338.31 to $378.60. However, notice that the marginal return of optimal customer equity decreases sharply with the preset promotion budget. For example, when the preset promotion budget increases from $20 to $30, the customer equity increases from $121.58 to $193.75 (i.e., increase of $72.17, or 59.4\%). However, when the preset promotion budget increases from $100 to $110, customer equity increases from $392.74 to $393.91 (i.e., increase of only $1.17, or 0.30\%). More importantly, Table 3 reveals that when the preset promotion budget approaches $110 per prospect, the customer equity no longer increases. This means that $393.91 per prospect is the highest customer equity the company can obtain for
the product, and that presetting the promotion budget for more than $110 per prospect leads only to wasting resources.

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Insert Table 3 about here
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Lastly, we present a sensitivity analysis for marginal contribution \( (m) \). The middle section of Table 3 shows that, as expected, customer equity goes up as unit marginal contribution increases. Indeed, for each $100 increase in marginal contribution, optimal customer equity increases slightly more than $100. However, we can also see from Table 3 that changes in marginal contribution have little impact on the optimal allocation of the preset promotion budget. To illustrate, when marginal contribution increases from $200 to $600 per purchase, optimal acquisition spending only decreases from $42.47 to $40.52 per prospect and the retention spending only increases from $53.61 to $59.81 per customer.

Finally, we conduct a sensitivity analysis with regard to discount rate \( (d) \). The bottom section of Table 3 indicates that as the discount rate increases, acquisition spending increases while retention spending decreases. For example, when the discount rate increases from 10% to 30%, optimal acquisition spending increases from $40.04 to $41.72 per prospect while optimal retention spending decreases from $61.66 to $55.32 per customer. Note that, whereas the optimal spending pattern changes only moderately (considering the tripling of the discount rate), optimal customer equity decreases more dramatically (from 433.32 to $285.22, or –34.2%).

7.2. Sensitivity analysis to inaccuracy of selected managerial inputs

Note that in the prototype real-world example above, the inputs from the manager are based on experience and subjective judgments and, therefore, may be inaccurate. Whereas the
preceding sensitivity analysis focuses on the effect of environmental parameters on the optimal outcomes, the sensitivity analysis in this section examines the effect of inaccuracy in the managerial inputs on the optimal outcomes.

Recall that the ceiling rates specify the acquisition and retention rates in the case where spending is unlimited. Thus, of the managerial inputs, we expect managers may have the most difficulty estimating the ceiling rates of acquisition and retention since these values are the most remote from the manager’s current situation (Chakravarti and Staelin, 1981). Thus, we conduct sensitivity analyses with regards to the inaccuracy of $C_a$ and $C_r$, as estimated by the manager for the current channel quality (i.e., $C_{a|\theta=6}$ and $C_{r|\theta=6}$), assuming all other inputs are accurate.

If the true values of $C_{a|\theta=6}$ and $C_{r|\theta=6}$ are different from the values the manager reports, the parameters defined in equations (4), (5), (8), and (9) take on different values. Thus, the optimal outcomes from the model will be changed, and, indeed, will be non-optimal. We begin with Table 4, which shows how the optimal results change with varying \emph{true} values of $C_{a|\theta=6}$ and $C_{r|\theta=6}$. To recall, the manager’s inputs indicate that $C_{a|\theta=6} = 0.60$ and $C_{r|\theta=6} = 0.65$. Assuming these are the correct values, the model indicates that optimal channel quality is $\theta = 2.8$, optimal acquisition and retention spending are $41.85 per prospect and $62.05 per customer, respectively, and optimal customer equity is $345.00. However, if, for example, $C_{a|\theta=6} = 0.55$ and $C_{r|\theta=6} = 0.60$ are the true ceiling rates at $\theta = 6$, the model would indicate that the optimal channel quality is actually $\theta = 3.4$ (with optimal acquisition spending and retention spending of $44.39 per prospect and $52.34 per customer, respectively). The optimal customer equity drops to $280.66. Table 4 provides optimal values for other combinations of $C_{a|\theta=6}$ and $C_{r|\theta=6}$. 
Note that, holding \( C_{r(\theta=6)} \) constant and allowing \( C_{a(\theta=6)} \) to increase, we observe an increasing optimal customer equity and a decreasing optimal channel quality. This is consistent with our expectation, because an increasing \( C_{a(\theta=6)} \) indicates an improved market environment and the potential to obtain, by whatever change in promotion budget allocation is optimally indicated, a higher customer equity. A similar observation can be made with regard to an increasing \( C_{r(\theta=6)} \).

We now examine the sensitivity of the model to inaccuracy in the manager’s ceiling rate estimates for the currently used channel. In this example, \( C_{a(\theta=6)} = 0.60 \) and \( C_{r(\theta=6)} = 0.65 \) are the values reported by the manager, which may or may not be the true values. The manager will observe the output of the model and choose channel quality \( \theta = 2.8 \), spend $41.85 per prospect in acquisition and $62.05 per customer in retention, and anticipate customer equity of $345.00. However, the maximum realized customer equity will be different, if the true ceiling rates are not as assumed. Table 5 reveals the respective customer equity values when the model uses \( C_{a(\theta=6)} = 0.60 \) and \( C_{r(\theta=6)} = 0.65 \), while the true values of \( C_{a(\theta=6)} \) and \( C_{r(\theta=6)} \) are the corresponding row and column designations.

Naturally, the suboptimal customer equities in Table 5 are smaller than their optimal cell-by-cell counterparts in Table 4, except for the middle cell, in which the manager’s estimates are accurate. Table 5 also presents the percentage differences between the suboptimal customer
equities and the corresponding optimal customer equities. In a sense, the percentage decreases represent the “punishment” for the inaccurate managerial inputs. For example, if the true values of \( C_{a|\theta=6} \) and \( C_{r|\theta=6} \) are 0.55 and 0.70, respectively, the optimal customer equity is $313.52, while the suboptimal customer equity is $291.46. Thus, the manager is “punished” by $22.06 (or 7.04%) for inaccurate assessments of the ceiling rates and subsequent implementation of a promotion budget allocation strategy corresponding to these inaccurate assessments.

Two other points in Table 5 are noteworthy. First, errors by the manager in estimating \( C_a \) and \( C_r \) in this example do not tend to “cancel out.” When estimation inaccuracy in \( C_a \) and \( C_r \) occurs simultaneously (in either the same or opposite direction), the suboptimal customer equity is significantly smaller than its counterpart based on the true values of \( C_{a|\theta=6} \) and \( C_{r|\theta=6} \). Second, the magnitudes of the percentage deviations in Table 5 are large enough to justify an optimization procedure (especially for larger target markets) but small enough so that the optimal solution is not overly sensitive to the manager’s estimation errors.

8. Discussion and limitations

In this paper, we extended the literature on customer equity optimization and optimal allocation of promotion budget spending (e.g., Berger and Nasr, 1998; Berger and Nasr-Bechwati, 2001; Blattberg and Deighton, 1996) in several ways. Specifically, we motivated channel quality as a relevant decision variable, explicated its role in the model, and demonstrated the existence of an optimal value. In addition, we relaxed the general concavity assumption between acquisition (retention) rate and acquisition (retention) spending by introducing the \( ADBUDG \) function (Little, 1970). This function does not mandate whether the acquisition (retention) response curve is concave or S-shaped. Rather, it allows the manager’s specific, real-
world situation to determine which shape is appropriate. Lastly, we removed the restriction embodied in previous models that zero expenditure on acquisition (retention) results in a zero acquisition (retention) rate.

After presenting an extended model, we demonstrated how to apply it by providing a prototype real-world example in which the decision calculus approach was used to estimate the model parameters. Also, we performed two types of sensitivity analyses. The first type of sensitivity analysis focused on how sensitive the optimal outcomes from the model are to the values of certain input parameters. The second type of sensitivity analysis illustrated how the optimal model outcomes are impacted by inaccuracy in the managerial inputs.

Our study, as any study, has limitations. The objective function in the model presumes that a vendor applies the same promotion channel in each purchase season and that customers make a purchase, if they are retained, only once during each purchase season. Additionally, the model only considers a one-company-one market case. More realistic is the case that, in each market segment, several companies may be competing with various products that may provide similar or complementary functions and benefits. These assumptions are, for the most part, shared by previous work (e.g., Berger and Nasr-Bechwati 2001). However, the model presented here should be able to accommodate a relaxation of nearly all of these assumptions.
References


## Table 1
Manager’s inputs to the extended model

<table>
<thead>
<tr>
<th>Inputs Requested from Manager</th>
<th>Parameter</th>
<th>Manager’s Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Size of the target market</td>
<td></td>
<td>1,000,000</td>
</tr>
<tr>
<td>2. Promotion budget allocated to the target market</td>
<td>$(1,000,000)B$</td>
<td>$60,000,000</td>
</tr>
<tr>
<td>3. Yearly contribution margin per customer</td>
<td>$m$</td>
<td>$400</td>
</tr>
<tr>
<td>4. Yearly discount rate</td>
<td>$d$</td>
<td>20%</td>
</tr>
<tr>
<td>5. Quality of the currently used promotion channel</td>
<td>$\theta$</td>
<td>6</td>
</tr>
<tr>
<td>6. Amount spent per prospect for the purpose of acquisition</td>
<td>$A$</td>
<td>$10</td>
</tr>
<tr>
<td>7. Amount spent per new customer for the purpose of retention</td>
<td>$R$</td>
<td>$20</td>
</tr>
<tr>
<td>8. Percentage of prospects the current promotion channel could have acquired if acquisition spending were unlimited</td>
<td>$C_{a</td>
<td>\theta=6}$</td>
</tr>
<tr>
<td>9. Percentage of prospects the current promotion channel could have acquired if there were no spending on acquisition</td>
<td>$a_0$</td>
<td>1%</td>
</tr>
<tr>
<td>10. Percentage of new customers that would have been retained if retention spending were unlimited</td>
<td>$C_{r</td>
<td>\theta=6}$</td>
</tr>
<tr>
<td>11. Percentage of new customers that would have been retained if there were no spending on retention</td>
<td>$r_0$</td>
<td>5%</td>
</tr>
<tr>
<td>12. Percentage of prospects purchasing product in past year</td>
<td>$a_{5</td>
<td>10}$</td>
</tr>
<tr>
<td>13. Percentage of prospects that would have purchased product if acquisition spending per prospect had been $10 higher</td>
<td>$a_{5</td>
<td>20}$</td>
</tr>
<tr>
<td>14. Percentage of new customers that have been retained</td>
<td>$r_{5</td>
<td>20}$</td>
</tr>
<tr>
<td>15. Percentage of new customers that would have been retained if the retention spending per new customer had been $10 higher</td>
<td>$r_{5</td>
<td>10}$</td>
</tr>
<tr>
<td>16. Percentage of prospects a totally quantity-oriented channel would have acquired if acquisition spending were unlimited</td>
<td>$C_{q</td>
<td>\theta=10}$</td>
</tr>
<tr>
<td>17. Percentage of new customers a totally quantity-oriented channel would have retained if retention spending were unlimited</td>
<td>$C_{r</td>
<td>\theta=10}$</td>
</tr>
</tbody>
</table>
Table 2
Sensitivity analysis for channel quality ($\theta$)

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$C_a$</th>
<th>$C_r$</th>
<th>$d^0$</th>
<th>$r^0$</th>
<th>$A^0$</th>
<th>$R^0$</th>
<th>$CE^0$</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>0.010</td>
<td>1.000</td>
<td>0.010</td>
<td>0.985</td>
<td>0.00</td>
<td>81.86</td>
<td>18.54</td>
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<td>1</td>
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<td>0.987</td>
<td>0.175</td>
<td>0.968</td>
<td>46.91</td>
<td>74.65</td>
<td>259.61</td>
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<tr>
<td>2</td>
<td>0.325</td>
<td>0.954</td>
<td>0.247</td>
<td>0.929</td>
<td>43.37</td>
<td>67.27</td>
<td>333.03</td>
</tr>
<tr>
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<td>...</td>
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<tr>
<td>2.7</td>
<td>0.384</td>
<td>0.920</td>
<td>0.287</td>
<td>0.890</td>
<td>42.00</td>
<td>62.68</td>
<td>344.91</td>
</tr>
<tr>
<td><strong>2.8</strong></td>
<td><strong>0.392</strong></td>
<td><strong>0.915</strong></td>
<td><strong>0.293</strong></td>
<td><strong>0.884</strong></td>
<td><strong>41.85</strong></td>
<td><strong>62.05</strong></td>
<td><strong>345.00</strong></td>
</tr>
<tr>
<td>2.9</td>
<td>0.399</td>
<td>0.909</td>
<td>0.298</td>
<td>0.878</td>
<td>41.71</td>
<td>61.43</td>
<td>344.78</td>
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<td>3</td>
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<td>0.903</td>
<td>0.303</td>
<td>0.871</td>
<td>41.58</td>
<td>60.82</td>
<td>344.28</td>
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<td>4</td>
<td>0.478</td>
<td>0.835</td>
<td>0.352</td>
<td>0.795</td>
<td>40.68</td>
<td>54.97</td>
<td>328.32</td>
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<tr>
<td>5</td>
<td>0.542</td>
<td>0.750</td>
<td>0.396</td>
<td>0.703</td>
<td>40.36</td>
<td>49.57</td>
<td>302.73</td>
</tr>
<tr>
<td>6</td>
<td>0.600</td>
<td>0.650</td>
<td>0.439</td>
<td>0.595</td>
<td>40.48</td>
<td>44.44</td>
<td>275.81</td>
</tr>
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<td>7</td>
<td>0.654</td>
<td>0.535</td>
<td>0.482</td>
<td>0.473</td>
<td>41.04</td>
<td>39.32</td>
<td>251.15</td>
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<tr>
<td>8</td>
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<td>0.404</td>
<td>0.527</td>
<td>0.337</td>
<td>42.21</td>
<td>33.75</td>
<td>230.40</td>
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<td>9</td>
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<td>0.259</td>
<td>0.579</td>
<td>0.190</td>
<td>44.75</td>
<td>26.36</td>
<td>215.14</td>
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</table>

Note: $d^0$ and $r^0$ are the optimal acquisition rate and retention rate, respectively. $A^0$ and $R^0$ are the optimal spending on acquisition and retention, respectively. $CE^0$ is the corresponding optimal customer equity. These notations apply to all tables. Also, $d = 0.20$ (i.e., 20%), $m = $400, and $B = $60.
Table 3
Sensitivity to preset budget ($B$), marginal contribution ($m$), and discount rate ($d$)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>$a^0$</th>
<th>$r^0$</th>
<th>$A^0$</th>
<th>$R^0$</th>
<th>$CE^0$</th>
<th>$\Delta CE$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.122</td>
<td>0.810</td>
<td>14.12</td>
<td>48.07</td>
<td>121.58</td>
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<td>30</td>
<td>0.192</td>
<td>0.815</td>
<td>20.54</td>
<td>49.31</td>
<td>193.75</td>
<td>72.17</td>
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<tr>
<td>40</td>
<td>0.249</td>
<td>0.822</td>
<td>27.10</td>
<td>51.72</td>
<td>255.19</td>
<td>61.44</td>
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<tr>
<td>50</td>
<td>0.294</td>
<td>0.829</td>
<td>33.92</td>
<td>54.68</td>
<td>303.15</td>
<td>47.96</td>
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<tr>
<td>60</td>
<td>0.328</td>
<td>0.835</td>
<td>41.04</td>
<td>57.83</td>
<td>338.31</td>
<td>35.16</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>0.353</td>
<td>0.840</td>
<td>48.49</td>
<td>60.92</td>
<td>362.70</td>
<td>24.39</td>
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<tr>
<td>80</td>
<td>0.372</td>
<td>0.844</td>
<td>56.27</td>
<td>63.80</td>
<td>378.60</td>
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<tr>
<td>90</td>
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<td>0.847</td>
<td>64.38</td>
<td>66.35</td>
<td>388.06</td>
<td>9.47</td>
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<tr>
<td>100</td>
<td>0.397</td>
<td>0.849</td>
<td>72.80</td>
<td>68.53</td>
<td>392.74</td>
<td>4.68</td>
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<tr>
<td>110</td>
<td>0.405</td>
<td>0.851</td>
<td>80.55</td>
<td>70.17</td>
<td>393.91</td>
<td>1.17</td>
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<tr>
<td>120</td>
<td>0.405</td>
<td>0.851</td>
<td>80.55</td>
<td>70.17</td>
<td>393.91</td>
<td>0.00</td>
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<tr>
<td>$m$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>200</td>
<td>0.333</td>
<td>0.824</td>
<td>42.47</td>
<td>52.61</td>
<td>123.65</td>
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<td>0.832</td>
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<td>55.97</td>
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<td>0.328</td>
<td>0.835</td>
<td>41.04</td>
<td>57.83</td>
<td>338.31</td>
<td>107.66</td>
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<td>500</td>
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<td>0.837</td>
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<td>446.24</td>
<td>107.93</td>
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<tr>
<td>600</td>
<td>0.326</td>
<td>0.839</td>
<td>40.52</td>
<td>59.81</td>
<td>554.32</td>
<td>108.07</td>
<td></td>
</tr>
<tr>
<td>$d$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10%</td>
<td>0.324</td>
<td>0.841</td>
<td>40.04</td>
<td>61.66</td>
<td>433.32</td>
<td></td>
<td></td>
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<tr>
<td>15%</td>
<td>0.326</td>
<td>0.838</td>
<td>40.60</td>
<td>59.51</td>
<td>377.94</td>
<td>-55.38</td>
<td></td>
</tr>
<tr>
<td>20%</td>
<td>0.328</td>
<td>0.835</td>
<td>41.04</td>
<td>57.83</td>
<td>338.31</td>
<td>-39.63</td>
<td></td>
</tr>
<tr>
<td>25%</td>
<td>0.329</td>
<td>0.833</td>
<td>41.41</td>
<td>56.46</td>
<td>308.49</td>
<td>-29.82</td>
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</tr>
<tr>
<td>30%</td>
<td>0.331</td>
<td>0.830</td>
<td>41.72</td>
<td>55.32</td>
<td>285.22</td>
<td>-23.28</td>
<td></td>
</tr>
</tbody>
</table>

Note: Unless parameter value is varied for purpose of sensitivity analysis, $\theta = 2.8$, $B = $60, $d = 0.20$ (i.e., 20%), and $m = $400.
Table 4
Sensitivity analysis for changes in $C_{r|\theta=6}$ and $C_{a|\theta=6}$

|                | $C_{a|\theta=6} = 0.55$ | $C_{a|\theta=6} = 0.60$ | $C_{a|\theta=6} = 0.65$ |
|----------------|--------------------------|--------------------------|--------------------------|
| $C_{r|\theta=6} = 0.60$ | $A^0$ 44.39              | $A^0$ 44.30              | $A^0$ 43.40              |
|                 | $R^0$ 52.34              | $R^0$ 54.44              | $R^0$ 54.84              |
|                 | $CE^0$ 280.66            | $CE^0$ 330.74            | $CE^0$ 398.26            |
|                 | $\theta^0$ 3.4           | $\theta^0$ 2.6           | $\theta^0$ 2.0           |
| $C_{r|\theta=6} = 0.65$ | $A^0$ 42.25              | $A^0$ 41.85              | $A^0$ 40.96              |
|                 | $R^0$ 60.69              | $R^0$ 62.05              | $R^0$ 62.43              |
|                 | $CE^0$ 298.15            | $CE^0$ 345.00            | $CE^0$ 404.59            |
|                 | $\theta^0$ 3.5           | $\theta^0$ 2.8           | $\theta^0$ 2.2           |
| $C_{r|\theta=6} = 0.70$ | $A^0$ 40.06              | $A^0$ 39.69              | $A^0$ 38.76              |
|                 | $R^0$ 67.07              | $R^0$ 68.63              | $R^0$ 68.30              |
|                 | $CE^0$ 313.52            | $CE^0$ 355.49            | $CE^0$ 406.34            |
|                 | $\theta^0$ 3.7           | $\theta^0$ 3.0           | $\theta^0$ 2.5           |

Note: $A^0$, $R^0$, and $CE^0$ are based on their corresponding optimal channel quality, $\theta^0$. 
Table 5
Sensitivity of suboptimal customer equity ($CE^{sub}$) to changes in $C_{r\theta=6}$ and $C_{a\theta=6}$

<table>
<thead>
<tr>
<th>$C_{r\theta=6}$</th>
<th>$CE^{\theta}$</th>
<th>$CE^{sub}$</th>
<th>$\Delta CE$</th>
<th>$CE^{\theta}$</th>
<th>$CE^{sub}$</th>
<th>$\Delta CE$</th>
<th>$CE^{\theta}$</th>
<th>$CE^{sub}$</th>
<th>$\Delta CE$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{a\theta=6} = 0.55$</td>
<td>280.66</td>
<td>270.15</td>
<td>-3.75%</td>
<td>330.74</td>
<td>326.01</td>
<td>-1.43%</td>
<td>398.26</td>
<td>382.49</td>
<td>-3.96%</td>
</tr>
<tr>
<td>$C_{a\theta=6} = 0.60$</td>
<td>298.15</td>
<td>286.25</td>
<td>-3.99%</td>
<td>345.00</td>
<td>345.00</td>
<td>0.00%</td>
<td>404.59</td>
<td>393.78</td>
<td>-2.67%</td>
</tr>
<tr>
<td>$C_{a\theta=6} = 0.65$</td>
<td>313.52</td>
<td>291.46</td>
<td>-7.04%</td>
<td>355.49</td>
<td>351.14</td>
<td>-1.22%</td>
<td>406.34</td>
<td>390.72</td>
<td>-3.85%</td>
</tr>
</tbody>
</table>

Note: Customer equities calculated with $A = $41.85, $R = $62.05, and $\theta = 2.8$, unless suboptimal allocation lead to violation of the budget constraint (in which case $R$ is reduced accordingly). Percentages represent ($CE^{sub} - CE^{\theta}$)/$CE^{\theta}$, where $CE^{\theta}$ is reproduced from Table 4 for convenience.
Fig. 1. Acquisition rate ($a$) and retention rate ($r$) rate as functions of promotion spending.
Fig. 2. Acquisition ceiling rate ($C_a$) and retention ceiling rate ($C_r$) as functions of channel quality ($\theta$).
Fig. 3. Optimal acquisition spending ($A^0$), retention spending ($R^0$), and customer equity ($CE^0$) as functions of channel quality ($\theta$).